M.S. Project), Vol.3, McGraw Hill, New York (1955).

SIMULTANEOUS SOLUTION OF THE EQUATIONS OF THE GRAVITATIONAL FIELD AND A FIELD OF NUCLEAR FORCES FOR A POINT SOURCE

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Numerical methods are used to solve the system of equations of the gravitational field and a nuclear field. The nuclear field is represented by a scalar meson field (of Yukawa type). It is found that the gravitational and nuclear field have a strong influence on each other: in the gravitational field, the singular Schwarzschild surface disappears, and the range of the scalar field increases considerably.

The problem of determining the gravitational field and nuclear field of a material point possessing mass and a nuclear charge is of great interest for elementary-particle physics (see [2]) as well as for relativistic astrophysics [1]. The problem has been solved [3] when the nuclear field is replaced by a massless scalar field. However, a massless scalar field cannot serve as a model for a field of nuclear forces, since it does not possess their main property – a short range. A scalar meson field (called in what follows simply a scalar field) does have this property when considered from the nonquantum point of view as a Yukawa field, and we therefore choose it as a representative of a field of nuclear forces to determine the gravitational and nuclear fields of a point source.

For the static spherically symmetric case in curvature coordinates, i.e., for the metric

$$ds^2=-A\,dr^2-r^2(d\theta^2+\sin^2\theta\,d\varphi^2)+Dc^2dt^2,$$

where A and D are functions of r alone (the remaining notation is standard), we find the energy–momentum tensor of the scalar field and write down Einstein's equations and the equation of the scalar field (ψ is the potential of the scalar field, μ is a constant of the order of 1 F, the prime denotes the derivative with respect to r, and G is the gravitational constant):

$$-\frac{1}{r}\frac{1}{A}\frac{D'}{D}+\frac{1}{r^2}\left(1-\frac{1}{A}\right)=\frac{G}{c^4}\left[-\frac{1}{A}(\psi')^2+\mu^2\psi^2\right],\quad \frac{1}{r}\frac{1}{A}\frac{A'}{A}+\frac{1}{r^2}\left(1-\frac{1}{A}\right)=\frac{G}{c^4}\left[\frac{1}{A}(\psi')^2+\mu^2\psi^2\right],$$

$$\psi''+\left(\frac{1}{2}\frac{D'}{D}-\frac{1}{2}\frac{A'}{A}+\frac{2}{r}\right)\psi'-\mu^2A\psi=0.$$

This system of equations does not admit exact analytic calculation, and we therefore solved it numerically. For convenience of the numerical solution, we introduce the variable $x = r/r_g$, where r_g is the gravitational radius of the source, and the new functions U and H:

$$\psi=\varepsilon\frac{c^2}{\sqrt{G}}U\exp(\mu r),\quad A=\frac{1}{H},$$

where ε is a constant whose value will be found below. The transformed equations (which are omitted, being cumbersome) contain, besides ε, the parameter $Q = \mu r_g$, which depends on the total mass of the source.

We formulate the boundary conditions. At infinity, where the mutual influence of the fields is weak (because they are weak), we must have a solution in the form of the Yukawa potential for the scalar field and the Schwarzschild solution for the gravitational field (there exists a detailed proof of the "Yukawa" asymptotic behavior at infinity [4]. A different asymptotic behavior [5] cannot be regarded as correct). We choose $\varepsilon=(q\sqrt{G})/(c^2r_g)$, where q is the scalar charge of the source, determined by the strength of the scalar field far from the source. The Yukawa potential then corresponds to $U = -(1/x)\exp(-2Qx)$ (the solution of the equation for U in the case of a "flat" metric), and the Schwarzschild solution to $D = H = 1 - 1/x$. These solutions of the equations at large values of the independent variable determine the boundary conditions at

Aviation Institute, Moscow. Translated from Teoreticheskaya i Matematicheskaya Fizika, Vol.47, No.1, pp.50–54, April, 1981. Original article submitted March 12, 1980.

 0040-5779/81/4701-0310$07.50

infinity.

A numerical solution was constructed by advancing from infinity, from large values of x to small ones. We used a difference method in conjunction with the Runge–Kutta method. Since the range of possible values of numbers on the computer is fairly restricted, this imposes limitations on the admissible values of the parameters ε and Q in the equation, and on the initial value x_0 of the independent variable. The calculations were made for $\varepsilon = 10$, $1 \leq Q \leq 30$, $x_0 \leq 30$. Q = 1 corresponds to a gravitational radius equal to the "normal" range $(1/\mu)$ of the scalar field. In this case, we find a strong influence of the scalar field on the gravitational field, which causes the disappearance of the singular Schwarzschild surface. The results obtained here correspond to regularity of the solution of the equations everywhere except the origin [6], and agree with the calculation of [7] for the case when the gravitational radius is of the same order of magnitude as the "normal" range of the scalar field.

The value Q = 30 corresponds to the case when the gravitational radius is 30 times greater than the range of the scalar field when considered without the gravitational field. Here, the influence of the gravitational field on the scalar field is more pronounced. The results for Q = 30 are shown in Table 1 and in Fig. 1. In Table 1, y_g is the Schwarzschild solution for g_{00}: $y_g = 1 - 1/x$; y_s is the solution of the equation for U in the case of flat metric: $y_s = -(1/x)\exp(-2Qx)$. Here, Ea stands for 10^a. In Fig. 1, the continuous curve is the curve $y = (-U)\exp(Qx)$ corresponding to the scalar potential ψ; the broken curve is the curve $y = (1/x)\exp(-Qx)$ corresponding to the Yukawa potential. The calculation reveals a sharp growth of the scalar potential at the gravitational radius. Here, the potential of the scalar field is 12 orders of magnitude greater than its normal ("Yukawa") value. The potential curve is deformed in such a way as to correspond to a field whose "physical" range exceeds the gravitational radius of the source. Thus, in a strong gravitational field the range of the scalar field is strongly increased. The scalar field is "extended," exhibiting an appreciable value outside the gravitational radius, beyond which it advances, being "smeared" over a much larger region of space than in the absence of a gravitational field.

Hitherto, it has not been clear what is responsible for the absence of a horizon in this metric – a formally mathematical or a physical reason. The exponential tail in the Yukawa potential ensures a vanishingly small but nonzero value of the scalar field arbitrarily far from the source. The absence of a horizon could be due solely to an unfortunate mathematical representation of the scalar field (short-range physically, but long-range from the formal mathematical point of view). By itself, therefore, this fact is not yet necessarily of interest from the physical point of view. But the numerical calculation does give it physical significance, since it shows for the first time that the absence of a horizon in the metric is inseparably related to the increase in the physical range of the scalar field. The gravitational field "stretches" the scalar field so strongly that in the region of the gravitational radius the scalar field reaches a very significant value. The gravitational field then ceases to be a field in vacuum and loses the horizon.

We emphasize that these conclusions about the influence of the gravitational field on the scalar field follow only from the results of our calculation. They cannot be drawn (and were not) on the basis of the previous papers [6-7].

Thus, we can say that a new physical effect has been established, namely, in a strong gravitational

TABLE 1

$x = r/r_g$	D/y_g	H/y_g	U/y_s
1,5	3,333E–1 3,333E–1	3,333E–1 3,333E–1	–5,463E–40 –5,463E–40
1,3	2,308E–1 2,308E–1	2,308E–1 2,308E–1	–1,763E–32 –1,026E–34
1,1	9,091E–2 9,091E–2	9,091E–2 9,091E–2	–4,050E–23 –1,973E–29
1,02	1,962E–2 1,961E–2	1,980E–2 1,961E–2	–3,345E–17 –2,586E–27
1,0	7,848E–3 0	3,813E–1 0	–1,745E–15 –8,756E–27
0,9	3,739E–3 –1,111E–1	5,947E+1 –1,111E–1	–1,341E–13 –3,925E–24
0,5	5,346E–4 –1,000E+0	6,522E+3 –1,000E+0	–6,416E–8 –1,871E–13

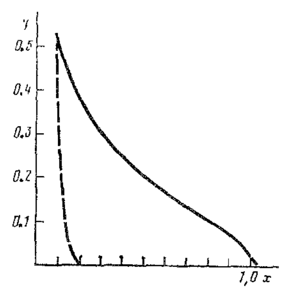

Fig. 1

field, the range of the scalar meson field (Yukawa field) increases strongly.

This effect must also be observed at $Q \sim 10^{18}$, when the gravitational radius and, therefore, the mass of the central body corresponds to a mass of an astronomical object. Indeed, in the investigated metric a horizon is always absent, i.e., for an arbitrarily large gravitational radius of the source [6]. It appears impossible that the mechanism of horizon destruction associated with the increase in the physical range of the scalar field should exist at fairly large values of the parameter Q but disappear at even larger values. It can be seen from the equation for the potential of the scalar field that the latter will always be strongly changed as the gravitational radius is approached. Indeed, if we assume that at large masses, when the gravitational radius is large (compared with the extremely short "normal" range of the scalar field), the scalar field is negligibly small near the gravitational radius, then D must be identical there to the Schwarzschild solution. But this solution gives

$$\frac{1}{2}\left(\frac{D'}{D}-\frac{A'}{A}\right)=\frac{D'}{D}=\frac{1}{x(x-1)}\frac{1}{r_g},$$

so that in the limit $x \to 1$ the "correction" introduced by the gravitational field into the equation of the scalar field becomes too large to be ignored. We conclude that the scalar meson field (Yukawa field) of a point source is stretched to distances not less than the gravitational radius of the source. It follows that if the gravitational radius is large the scalar field must be transformed from a short- to a long-range field.

It must be borne in mind that the numerical calculations here were made for a very small value of the parameter ε. Our choice of ε was made to permit numerical solution of the problem by the indicated method on the computer at our disposal. The chosen value $\varepsilon = 10$ corresponds to a small ratio α of the scalar charge of the complete central body to its mass: $\alpha = 5.166 \cdot 10^{-3}$. But if we have in mind astronomical objects such as neutron stars, the scalar field (Yukawa field) must be regarded as a model of the nuclear field produced by the neutrons. Then α must be of the order of the ratio of the nuclear charge to the mass for the neutron, for which $\alpha \sim 10^{14}$. Then ε will be 16 orders of magnitude greater than the value adopted here. The energy of the scalar field for such a large charge-to-mass ratio must also be very large and, as we may say, there is something "to be stretched" by the influence of the gravitational field. The effect must be much greater than in the calculated case.

The influence exerted by the gravitational field on all other fields must be universal. If the short-range field is "stretched" by the gravitational field, so must any other short-range field. Irrespective of the appropriateness of simulating the nuclear field by a scalar field, we can expect a short-range nuclear field to be transformed into a long-range field in the same way as the short-range scalar field. On the gravitational collapse of objects such as neutron stars, the nonvanishing (by virtue of the conservation of the baryon charge) nuclear field must be "stretched" to the gravitational radius even when the surface of the star has already passed below the gravitational radius. The nuclear field that extends beyond the gravitational radius precludes the formation of a horizon in the metric of a collapsed object. Therefore, the existence of black holes appears impossible. In other words, the possibility of transforming short-range force fields into long-range fields (under the conditions discussed above) rules out the possibility of formation of black holes. The existence of the effect by which the range of force fields is changed in the gravitational field must lead to interesting results in relativistic astrophysics.

LITERATURE CITED

1. Albert Einstein and the Theory of Gravitation. A Collection of Papers [Russian translation], Mir, Moscow (1979).
2. M. A. Markov, "Global properties of matter in the collapsed state (black holes)," Usp. Fiz. Nauk, **111**, 3 (1973).
3. I. Z. Fisher, "Scalar mesostatic field with allowance for gravitational effects," **18**, No. 7, 636 (1948) (Journal title not given in the Russian original).
4. G. Stephenson, "A static spherically symmetric solution of the Einstein–Maxwell–Yukawa field equations," Proc. Cambridge Philos. Soc., **58**, 521 (1962).
5. R. A. Asanov, "Static scalar and electric fields in Einstein's theory of gravitation," Zh. Eksp. Teor. Fiz., **53**, 673 (1967).
6. A. A. Solodov, "On the external fields of collapsed bodies," Teor. Mat. Fiz., **24**, 136 (1975).
7. R. A. Asanov, "On a point source of a scalar field with mass in the theory of gravitation," Teor. Mat. Fiz., **20**, 66 (1974).